GCSE Maths - Vectors Questions

1. $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$AB : BC = 2 : 5 \Rightarrow \overrightarrow{AB} = \frac{2}{7}\overrightarrow{AC}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -(2\underline{a} + \underline{b}) + 3\underline{a} + 2\underline{b} = \underline{a} + \underline{b}$

$\Rightarrow \overrightarrow{AC} = \frac{7}{2}\overrightarrow{AB} = \frac{7}{2}(\underline{a} + \underline{b})$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 2\underline{a} + \underline{b} + \frac{7}{2}(\underline{a} + \underline{b})$

$= \frac{11}{2}\underline{a} + \frac{9}{2}\underline{b}$

2. (a) $\overrightarrow{SQ} = \underline{a} - \underline{b}$

(b) $\overrightarrow{NQ} = \frac{2}{5}\overrightarrow{SQ} = \frac{2}{5}(\underline{a} - \underline{b})$

$\overrightarrow{NR} = \overrightarrow{NQ} + \overrightarrow{QR} = \frac{2}{5}(\underline{a} - \underline{b}) + \underline{b} = \frac{2}{5}\underline{a} - \frac{2}{5}\underline{b} + \underline{b}$

$= \frac{2}{5}\underline{a} + \frac{3}{5}\underline{b}$

3. (a)(i) $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS} = \underline{b} - \underline{a}$

(ii) $\overrightarrow{QY} = \frac{1}{2}\underline{b} - \underline{a}$

(iii) $\overrightarrow{PX} = \underline{a} + \frac{1}{2}(\underline{b} - \underline{a}) = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$

(b) $\overrightarrow{PX} = \frac{3}{2}\overrightarrow{PR} = \frac{3}{2}\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}6\\3\end{pmatrix}$

$P = (3, 1)$ $\quad X = (3+6, 1+3) = (9, 4)$

$\overrightarrow{XV} = \begin{pmatrix}-5\\4\end{pmatrix}$ $\quad V = (9-5, 4+4)$

$\Rightarrow V = (4, 8)$

4. $\vec{CB} = 4\underline{a}$ $\quad CB : BX = 2 : 3$

$\therefore \vec{BX} = 6\underline{a}$

$\vec{AX} = \vec{AB} + \vec{BX} = 3\underline{c} + 6\underline{a}$

$\vec{CY} = 2\vec{AX} = 6\underline{c} + 12\underline{a}$

$\therefore \vec{OY} = \vec{OC} + \vec{CY} = 3\underline{c} + 6\underline{c} + 12\underline{a}$

$= 9\underline{c} + 12\underline{a}$

5. $\vec{OM} = \underline{a} + \frac{1}{2}\underline{b}$ $\qquad \vec{AM} = \frac{1}{2}\underline{b}$

$\vec{MX} = \frac{1}{2}\underline{b} + \underline{a}$

$\vec{OX} = \underline{a} + \frac{1}{2}\underline{b} + \frac{1}{2}\underline{b} + \underline{a} = 2\underline{a} + \underline{b} = 2\vec{OM}$

$\therefore$ vectors are parallel and on a straight line