

A-Level Math - Year 1 Integration

$$\begin{aligned} 1. \quad & \int (2x^5 - \frac{1}{4}x^{-3} - 5) dx \\ & = \frac{2x^6}{6} - \frac{\frac{1}{4}x^{-2}}{-2} - 5x + C \\ & = \frac{x^6}{3} + \frac{1}{8x^2} - 5x + C \end{aligned}$$

$$\begin{aligned} 2. \quad & \int (10x^4 - 4x - 3x^{-1/2}) dx \\ & = \frac{10x^5}{5} - \frac{4x^2}{2} - \frac{3x^{1/2}}{1/2} + C \\ & = 2x^5 - 2x^2 - 6\sqrt{x} + C \end{aligned}$$

$$\begin{aligned} 3. (a) \quad & \int 10x(x^{\frac{1}{2}} - 2) dx = \int (10x^{3/2} - 20x) dx \\ & = \frac{10x^{5/2}}{5/2} - \frac{20x^2}{2} + C = 4x^{5/2} - 10x^2 + C \end{aligned}$$

$$\begin{aligned} (b) \quad & \int_0^4 10x(x^{\frac{1}{2}} - 2) dx = [4x^{5/2} - 10x^2]_0^4 \\ & = 4 \times 4^{5/2} - 10 \times 16 - 0 \\ & = 128 - 160 = -32 \end{aligned}$$

$$\begin{aligned} & \int_4^9 10x(x^{\frac{1}{2}} - 2) dx = [4x^{5/2} - 10x^2]_4^9 \\ & = 4 \times 9^{5/2} - 10 \times 81 - (-32) \\ & = 972 - 810 + 32 = 194 \end{aligned}$$

$$\therefore \text{total area} = 194 + 32 = 226.$$

$$4. (a) y = 4x^3 + 9x^2 - 30x - 8$$

$$\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$$

$$2x^2 + 3x - 5 = 0$$

$$(2x+5)(x-1) = 0$$

$$x = -\frac{5}{2} \quad \text{or} \quad x = 1$$

↖ outside of range

∴ $x = 1$ at A

$$(b) \quad x = 1 \quad y = 4 + 9 - 30 - 8 = -25$$

$$\text{triangle AB} = \frac{25 \times 1}{2} = 12.5$$

$$\int_{-\frac{1}{4}}^1 (4x^3 + 9x^2 - 30x - 8) dx$$

$$= [x^4 + 3x^3 - 15x^2 - 8x]_{-\frac{1}{4}}^1$$

$$= 1 + 3 - 15 - 8 - (3.90625 \times 10^{-3})$$

$$= -0.046875 - 0.9375 + 2$$

$$= -17.98 \dots$$

$$\therefore \text{total area} = 17.98 + 12.5 = 30.48 \text{ (to 2dp)}$$